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Coordinate systems for solar image data

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ABSTRACT

A set of formal systems for describing the coordinates of solar image data is proposed. These systems build on current practice in applying coordinates to solar image data. Both heliographic and heliocentric coordinates are discussed. A distinction is also drawn between heliocentric and helioprojective coordinates, where the latter takes the observer's exact geometry into account. The extension of these coordinate systems to observations made from non-terrestial viewpoints is discussed, such as from the upcoming STEREO mission. A formal system for incorporation of these coordinates into FITS files, based on the FITS World Coordinate System, is described, together with examples.

Key words. standards – sun: general – techniques: image processing – astronomical data bases: miscellaneous – methods: data analysis

1. Introduction

Solar research is becoming increasingly more sophisticated. Advances in solar instrumentation have led to increases in spatial resolution, and will continue to do so. Future space missions will view the Sun from different perspectives than the current view from ground-based observatories, or satellites in Earth orbit. Both of these advances will require more careful attention to the coordinate systems used for solar image data. In fact, some taste of this has already occured with the Solar and Heliospheric Observatory (SOHO) satellite (Domingo et al. 1995), which views the Sun from the inner Lagrange point between Earth and the Sun. The Sun appears approximately 1% bigger from SOHO than it does from Earth, requiring adjustment whenever SOHO images are compared with data from ground-based observatories, or satellites in low Earth orbit.

Although there is widespread agreement on the coordinate systems to be used for interplanetary space (Russell 1971; Hapgood 1992; Fränz & Harper 2002) no formal structure exists for solar image coordinates, except for the well-established heliographic coordinate systems. In particular, there is no agreement on how these coordinates should appear in FITS headers, with potential confusion when data from one observatory is compared to data from another. This document outlines the various possible coordinate systems which may be used for solar image data, and to show how these coordinate systems relate to the World Coordinate System (WCS) formalism used in FITS files. In devising the coordinate systems outlined below, attention is given to current practice within the solar imaging community.

Normal coordinate systems used for extra-solar observations—such as right ascension and declination, or galactic longitude and latitude—only need to worry about two spatial dimensions. The same can be said for normal cartography of a planet such as Earth. However, to properly treat the complete range of solar phenomena, from the interior out into the corona, a complete three-dimensional coordinate system is required. Unfortunately, not all the information necessary to determine the full three-dimensional position of a solar feature is usually available. Therefore, in developing coordinate systems for solar data, one must take into account that one of the axes of the coordinate system may be missing.

Also, since the Sun is a gaseous body, there are no fixed points of reference on the Sun. What's more, different parts of the Sun rotate at different rates. The rotation rate depends not only on latitude, but also on how deeply the magnetic field lines of a given feature are anchored in the photosphere. Thus, for example, active regions follow different differential rotation laws than smaller-scale magnetic features at the same latitudes.

For the above reasons, attention will be given to coordinate systems which take the observer's viewing geometry into consideration. Some coordinate systems will be rotating with respect to other coordinate systems, and in all the coordinate systems, at least some part of the Sun will be moving relative to that coordinate system. For those reasons, the coordinate system should not really be considered complete without also taking time into account. In the discussion which follows, all coordinates will assume a given observation time t.

With solar imaging instrumentation now planned for spacecraft which will operate at large distances away from the Earth-Sun line, such as STEREO (Socker et al. 1996), consideration must also be given as to how the viewpoint of the instrument should be taken into account in the coordinate system. This is discussed in Sec. 9.1.

1.1. Coordinate Systems within FITS files

Two different styles of including coordinate information within FITS files will be considered here. The first is the coordinate system from the original FITS specification, while the second is the more modern World Coordinate System. The latter allows for much more flexibility in the types of coordinate systems which can be expressed, as well as in describing how the instrument coordinate axes map into real-world coordinates. For those reasons, we will concentrate on the World Coordinate System implementations of the coordinate systems. However, the older system will also be considered where appropriate.

1.1.1. The Original FITS Coordinate System

In the original FITS paper (Wells et al. 1981), the coordinates of a data array were specified with the following keywords in the FITS header:

`CRPIX`*n*: A reference pixel, which is subtracted from the pixel coordinates along the n axis. This can be a fractional value, representing a point between pixel centers, and can even lie outside the array. Common values are the first pixel `CRPIX`*n*=1 or the center pixel `CRPIX`*n*=(`NAXIS`*n*+1)/2. Note that `CRPIX`*n* follows the Fortran convention of counting from 1 to N, instead of from 0 to $N-1$ as is done in some programming languages.

`CRVAL`*n*: The coordinate value of the reference pixel along axis n.

`CDELT`*n*: The pixel spacing along axis n.

`CROTA`*n*: The rotation angle, in degrees, to apply to axis n. Normal usage is to associate both spatial axes with the same rotation angle, while any other axes of the array would have a rotation of zero. The exact method of applying the rotation angle was not specified in the original paper, but commonly used conventions have developed over the years, as discussed in Sec. 8.

`CTYPE`*n*: A string value labeling each coordinate axis. A common system, used by the SOHO spacecraft, labels the westward axis as `SOLARX` and the northward axis as `SOLARY`.

In the simple case where the rotation angle is zero, the transformation from pixel to real world coordinates is then

$$x = \mathtt{CRVAL}n + \mathtt{CDELT}n \times (i - \mathtt{CRPIX}n).$$

(The case where `CROTA`*n* $\neq 0$ is discussed in Sec. 8.) Later, we will show that this older system can be considered a special case of the coordinate systems described below.

1.1.2. The World Coordinate System

The World Coordinate System (WCS) (Greisen & Calabretta 2002; Calabretta & Greisen 2002) is a system for describing the coordinates of a FITS array, and includes a system of describing various map projections for spherical coordinates.[1] In simplified form, the basic process is as follows:

1. The pixel coordinates of an image, which in the FITS convention run from 1 to N, are converted into projection plane coordinates in physical units, using parameters described by the following keywords:

   `CRPIX`*j*: Has the same definition as in the original FITS coordinate system. For spherical coordinates, there may be certain restrictions on the reference pixel.

   `PC`*i_j*: A coordinate transformation matrix, describing the conversion from pixel coordinate axes j into intermediate pixel coordinates i. The default values for `PC`*i_j* are 1 for $j = i$ and 0 elsewhere. This transformation matrix replaces the older `CROTA`*n* keyword, which is now considered obsolete in the WCS formalism. As well as rotations, the `PC` matrix can also encode other transformations, such as skew.

   `CDELT`*i*: Has essentially the same definition as in the original FITS coordinate system. In the WCS formalism, `CDELT`*i* is applied *after* the `PC`*i_j* coordinate transformation.

   (There is also an optional form of the WCS formalism in which both the `PC`*i_j* and `CDELT`*i* keywords are combined into a single `CD`*i_j* matrix. This is discussed in more detail in Sec. 1.2.)
2. For spherical coordinates, the projection plane coordinates are converted into an intermediate spherical coordinate system using the following keywords:

   `CTYPE`*i*: A string value representing the type of each coordinate axis. For spherical coordinates, the first four characters express the coordinate type, while the second four characters express the map projection used. (For non-spherical coordinates, the second four characters can also express a non-linear transformation such as "`-LOG`".)

   `PV`*i_m*: Additional parameters required in some coordinate systems.
3. The intermediate coordinate system is converted into the standard coordinate system using the following keywords:

   `CRVAL`*i*: The coordinate value of the reference pixel along axis i.

   `CUNIT`*i*: The units of the coordinates along axis i.

   `LONPOLE`, `LATPOLE`: Angular coordinates used for spherical transformations. Only used for spherical coordinates. These keywords take default values depending on the map projection used, if not present in the FITS header.

For linear coordinates, steps 2 and 3 are combined into a single step.

[1] Future planned developments for the WCS include spectral coordinates, non-linear distortions, and possibly temporal coordinates.

Table 1. Alternate forms of the World Coordinate systems for use in binary tables. The parameters *i*, *j*, and *m* are integer indices, while *a* represents an optional single-character code for alternate coordinate systems, as explained in the text. The additional parameter *n* represents the column in the binary table.

Keyword description	Main array	Binary tables Primary	Alternate
Axis type	`CTYPE`*ia*	*i*`CTYP`*n*	*i*`CTY`*na*
Axis units	`CUNIT`*ia*	*i*`CUNI`*n*	*i*`CUN`*na*
Reference value	`CRVAL`*ia*	*i*`CRVL`*n*	*i*`CRV`*na*
Pixel scale	`CDELT`*ia*	*i*`CDLT`*n*	*i*`CDE`*na*
Reference point	`CRPIX`*ja*	*j*`CRPX`*n*	*j*`CRP`*na*
Transformation matrix	`PC`*i_ja*	*ij*`PC`*n*	*ij*`PC`*na*
Alt. Trans. matrix	`CD`*i_ja*	*ij*`CD`*n*	*ij*`CD`*na*
Coordinate parameter	`PV`*i_ma*	*i*`V`*n_m*	*i*`V`*n_ma*
Native longitude	`LONPOLE`*a*	`LONP`*n*	`LONP`*na*
Native latitude	`LATPOLE`*a*	`LATP`*n*	`LATP`*na*

It is a requirement in FITS files, and in WCS in particular, that all angles must be in degrees. (However, see Sec. 9.2.) For simplicity, in the following, all trigonometric functions are assumed to have their arguments in degrees, while the inverse functions are assumed to return values in degrees.

It is possible in WCS to have more than one coordinate system for any given data set. An alternate coordinate system would be expressed using all the above keywords followed by one of the letters "A" to "Z", e.g. `CRPIX1A`, `PC1_1A`, etc. The keywords WCSNAME*a* can be used to label each alternative coordinate system—see Figs. 5 to 7.

Modified versions of the above keywords for use in binary tables are listed in Greisen & Calabretta (2002) and in Calabretta & Greisen (2002). For convenience, these are reproduced in Table 1. For example, the keyword `CRPIX3` in the main part of a FITS file would translate into `3CRPX7` for an array stored in the seventh column of a FITS binary table, while `CRPIX3A` would translate into `3CRP7A`. (Also, see the above references for a discussion of pixel lists.)

1.2. The alternate `CD` transformation matrix

The World Coordinate System has undergone considerable evolution in its development. The main impetus for this effort was to remove the ambiguity in the use of the `CROTA`*i* keyword, and to allow more flexibility in the kinds of coordinate transformations that could be applied to data. Part of the process in developing the WCS formalism was deciding whether or not the pixel spacing parameters `CDELT`*i* should be kept separate, or merged into the overall transformation. Some earlier drafts of Greisen & Calabretta (2002) (and earlier presentations of the present work) deprecated the use of the `CDELT`*i* keyword, using instead a single transformation matrix `CD`*i_j* which incorporated the functions of both `CDELT`*i* and `CROTA`*i*. However, the final version of the WCS paper has both the `CDELT`*i*, `PC`*i_j* and the `CD`*i_j* formalisms. There are thus two ways that coordinate transformations can be entered into FITS files:

- One can either use the pixel spacing keywords `CDELT`*i*, together with the (dimensionless) transformation matrix `PC`*i_j*,
- Or, one can combine both of these functions into the single (dimensioned) transformation matrix `CD`*i_j*.

The two alternate formats are related through the expression

$$\mathtt{CD}i\_j = \mathtt{CDELT}j \times \mathtt{PC}i\_j.$$

Either format is acceptable, but only one should be used in any FITS file. For compatibility with older FITS formats, the `CDELT`*i*, `PC`*i_j* formalism is recommended.

2. Heliographic Coordinates

The well-known heliographic[2] coordinate system expresses the latitude Θ and longitude Φ of a feature on the solar surface, and can be extended to three dimensions by adding the radial distance r from the center of the Sun. The rotational axis used to define the coordinate system is based on the original work of Carrington (1863). Seidelmann et al. (2002) lists the right ascension and declination of the solar north pole as $\alpha_0 = 286\overset{\circ}{.}13$, $\delta_0 = 63\overset{\circ}{.}87$. There are two basic variations on the heliographic system, which we will refer to as Stonyhurst and Carrington heliographic. Both use the same solar rotational axis, and differ only in the definition of longitude.

There are several limitations to the use of heliographic coordinates when used with two-dimensional image data:

- Without the r axis, coordinates can only be expressed for pixels on the solar disk.
- Features which are significantly elevated above the solar surface will project into coordinates (Θ, Φ) which are different from their true coordinates in a complete (r, Θ, Φ) system.
- At some wavelengths, particularly in Helium lines, the apparent solar limb will be larger than $R_\odot$, by as much as 1.5%.

However, the coordinate system is well suited for many types of solar data.

2.1. Stonyhurst Heliographic Coordinates

The origin of the Stonyhurst heliographic coordinate system is at the intersection of the solar equator and the central meridian as seen from Earth. Thus, the coordinate system remains fixed with respect to Earth, while the Sun rotates (synodically) underneath it. The angles Θ and Φ are given in degrees, with Θ increasing towards solar North, and Φ increasing towards the solar West limb. The distance r is either a physical distance in meters, or is relative to the solar photospheric diameter $R_\odot \approx 6.96 \times 10^8$ m. This coordinate system is demonstrated in Fig. 1.

[2] Since the Sun is assumed to have zero oblateness, and rotates in the prograde sense, there is no distinction between planetographic and planetocentric latitudes and longitudes for the Sun. We will use the traditional term heliographic when referring to latitude and longitude.

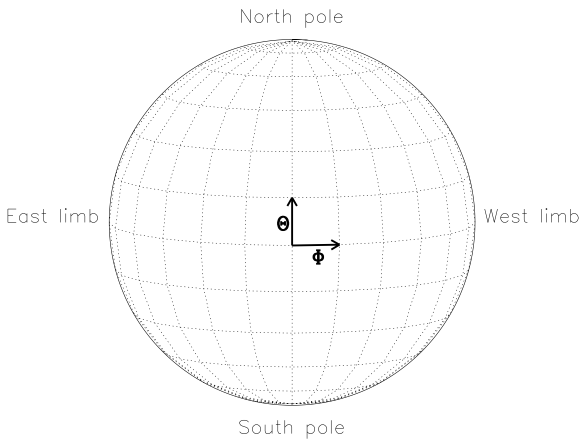

Fig. 1. A diagram of the Sun, showing lines of constant Stonyhurst heliographic longitude and latitude on the solar disk. The origin of the coordinate system is at the intersection of the solar equator and the (terrestrial) observer's central meridian. This representation is also known as a Stonyhurst grid.

An alternative to the r coordinate is the height $h = r - R_\odot$ relative to the solar surface, where h is positive above the surface and negative below the surface.

It should be noted that Stonyhurst heliographic coordinates are closely related to Heliocentric Earth Equatorial (HEEQ) coordinates (Hapgood 1992). Conversion between these two systems are given by the equations[3]

$$\begin{aligned} r &= \sqrt{X^2_{\mathrm{HEEQ}} + Y^2_{\mathrm{HEEQ}} + Z^2_{\mathrm{HEEQ}}}, \\ \Theta &= \tan^{-1}\left(Z_{\mathrm{HEEQ}} / \sqrt{X^2_{\mathrm{HEEQ}} + Y^2_{\mathrm{HEEQ}}}\right), \\ \Phi &= \arg(X_{\mathrm{HEEQ}}, Y_{\mathrm{HEEQ}}), \end{aligned} \tag{1}$$

$$\begin{aligned} X_{\mathrm{HEEQ}} &= r\cos\Theta\cos\Phi, \\ Y_{\mathrm{HEEQ}} &= r\cos\Theta\sin\Phi, \\ Z_{\mathrm{HEEQ}} &= r\sin\Theta. \end{aligned} \tag{2}$$

To maintain this relation, and to avoid confusion, we suggest that for non-terrestrial observers, the origin should still be referenced to the central meridian as seen from Earth (specifically geocenter). Thus, the coordinates of any feature on the surface at a given time t will be the same for any observer. Observations made from a distance significantly different than 1 A.U. may require a light travel time correction for utmost accuracy.

2.2. Carrington Heliographic Coordinates

The Carrington coordinate system is a variation of the heliographic system which rotates at an approximation to the mean solar rotational rate, as originally used by Carrington (1863). The sidereal period of the Carrington system is 25.38 days, which translates into a mean synodic period of 27.2753 days (Stix 1989). Seidelmann et al. (2002) gives the angle of the prime meridian from the ascending node as 84.°10 at J2000.0. Each time the Carrington prime meridian coincides with the central meridian as seen from Earth, which happens once each 27.21 to 27.34 days, depending on where Earth is in its orbit, marks the beginning of a new "Carrington rotation". These rotations are numbered sequentially, with Carrington rotation number 1 commencing on 9 November 1853. The start date and time of each Carrington rotation (also known as the "synodic rotation number") is published in the *Astronomical Almanac*. For example, Carrington rotation number 1900 began on 2 September 1995.

Carrington longitude is offset from Stonyhurst longitude by a time-dependent scalar value. In other words, at any given time t, the relationship between Stonyhurst longitude Φ and Carrington longitude Φ_{C} is given by the equation

$$\Phi_{\mathrm{C}} = \Phi + L_0, \tag{3}$$

where L_0 is the Carrington longitude of the central meridian as seen from Earth. At the start of each Carrington rotation, $L_0 = 360°$ and steadily *decreases* until it reaches $L_0 = 0$, at which point the next Carrington rotation starts. Note that this is the opposite behavior of the decimal Carrington number of the central meridian, which constantly increases. The daily values of L_0 are available in the *Astronomical Almanac*.

3. Heliocentric Coordinates

Heliocentric coordinates express the true spatial position of a feature in physical units from the center of the Sun. There are a number of well-established heliocentric coordinate systems used in space physics (Hapgood 1992). Examples include Heliocentric Aries Ecliptic (HAE), Heliocentric Earth Ecliptic (HEE), and Heliocentric Earth Equatorial (HEEQ). Each of these coordinate systems consists of three mutually perpendicular axes, X, Y, and Z, which together form a right-handed coordinate system. For example, the HEE system has an X axis pointing along the Sun-Earth line, and a Z axis pointing along the ecliptic north pole. (A precise definition of the ecliptic can be found in Lieske et al. (1977).) In this work, we will define additional heliocentric coordinate systems oriented specifically towards solar image data.

No solar observation from a single perspective can truly be said to be in heliocentric coordinates as defined below. Lines of sight from the observer to the Sun are not truly parallel to each other, so that translating image position into a physical distance will depend to some extent on where the feature is along the line of sight. Also, no attempt is made to distinguish the various possible map projections—e.g. whether one is measuring an angular distance, the sine of the angle, or the tangent of the angle. However, for many cases, these distinctions are unimportant, and heliocentric coordinates as defined here are often very useful. They also serve as a basis for the helioprojective coordinate systems discussed later.

No matter what perspective an observer has, the heliocentric coordinate systems discussed below will have axes defined

[3] The function arg is defined such that $\arg(x, y) = \tan^{-1}(y/x)$, where the answer can lie anywhere from -180° to 180°, and thus resolves the the ambiguity between which quadrant the result should lie in.

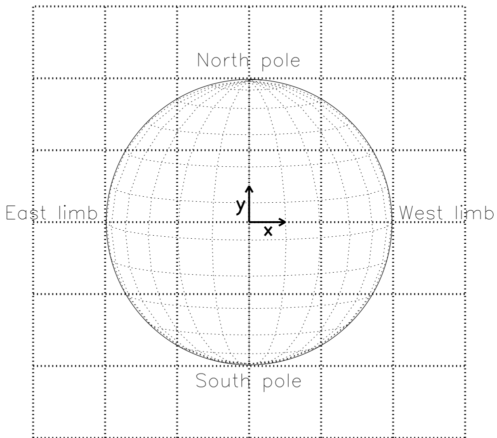

Fig. 2. A diagram of the Sun, with lines of constant heliocentric-cartesian position (x, y) overlayed. The z axis points out of the page.

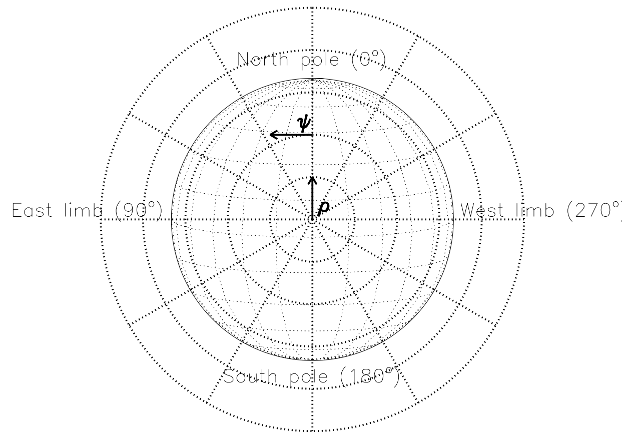

Fig. 3. A diagram of the Sun demonstrating heliocentric-radial coordinates, with lines of constant impact parameter ρ and position angle ψ overlayed. The value of ψ at each of the four compass points is also shown. The z axis points out of the page.

relative to that observer. Thus, unlike the heliographic case, an observation made from a non-terrestrial platform will measure coordinates which will be different from those measured from Earth. Hence, at least for non-terrestrial observations, information must also be provided about the observers position to properly define the coordinate system (see Sec. 9.1).

The class of observer-dependent heliocentric coordinate systems discussed in this report falls into two subcategories: heliocentric-cartesian and heliocentric-radial.

3.1. Heliocentric-Cartesian Coordinates

This is a true (x, y, z) Cartesian coordinate system, with each of the axes being perpendicular to each other, and with all lines of constant x (or y or z) being parallel to each other. The z axis is defined to be parallel to the observer-Sun line, pointing toward the observer. The y axis is defined to be perpendicular to z and in the plane containing both the z axis and the solar North pole axis, with y increasing towards solar North. The x axis is defined to be perpendicular to both y and z, with x increasing towards solar West. Thus, it is a right-handed coordinate system. Each axis is expressed as either a physical distance in meters, or relative to $R_\odot$. The heliocentric-cartesian coordinate system is demonstrated in Fig. 2.

Heliocentric-cartesian coordinates are also known as heliocentric (or heliographic) Radial-Tangential-Normal (HGRTN) coordinates (Fränz & Harper 2002), except with different nomenclature. The heliocentric-cartesian axes x, y, z are equivalent to the HGRTN axes Y_{HGRTN}, Z_{HGRTN}, and X_{HGRTN} respectively.

3.2. Heliocentric-Radial Coordinates

Heliocentric-radial coordinates share the same z axis as heliocentric-cartesian coordinates, but replace (x, y) with (ρ, ψ). The parameter ρ is the radial distance from the z axis, and is also known as the impact parameter. It is expressed either as a physical distance or relative to $R_\odot$. Surfaces of constant ρ form cylinders. The position angle ψ is measured in degrees counterclockwise from the projection of the solar North pole. This coordinate system is demonstrated in Fig. 3.

4. Projected Coordinate Systems

Data taken from a single perspective can only approximate true heliocentric coordinates. A more precise rendition of coordinates should recognize that observations are projected against the celestial sphere. This class of coordinate systems, which we will call helioprojective, mimics the heliocentric coordinate systems discussed above, replacing physical distances with angles. Although there's a one-to-one correspondence between the heliocentric parameters and the helioprojective parameters, the latter is really an observer-centric system. When the observer is on or just above Earth, these can be described as geocentric coordinate systems.

All projective angles discussed here have an origin at disk center. This is the apparent disk center as seen by the observer, without any corrections made for light travel time or aberrations. Such considerations do not play a role in these solar-specific coordinate systems.

Because helioprojective coordinates are ultimately spherical in nature, the full map-projection aspects of WCS come into play. Thus, one must take into account exactly what map projections are used in taking the data.

4.1. Helioprojective-Cartesian Coordinates

This is the projected equivalent of heliocentric-cartesian coordinates, where the distance parameters x and y are replaced with the angles θ_x and θ_y. Close to the Sun, where the small angle approximation holds, the heliocentric-cartesian and helioprojective-cartesian are related through the equations

$$x \approx d\left(\frac{\pi}{180^\circ}\right)\theta_x \approx D_\odot\left(\frac{\pi}{180^\circ}\right)\theta_x, \tag{4}$$

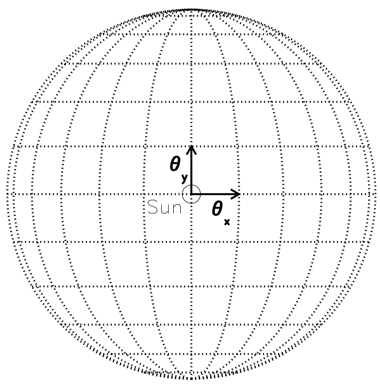

Fig. 4. The helioprojective-cartesian coordinate system extended outward to a full celestial hemisphere. The lines of constant θ_x and θ_y are drawn as if looking straight down into the interior surface of a hemispherical bowl—in the WCS this is known as a `SIN` projection. Only in an area close to the Sun (shown not to scale) is the coordinate system close to Cartesian.

$$y \approx d(\frac{\pi}{180^\circ})\theta_y \approx D_\odot(\frac{\pi}{180^\circ})\theta_y,$$

where d is the distance between the observer and the feature, and $D_\odot$ is the distance between the observer and Sun center. At farther distances, where the distinction between distance and angle becomes more important, the definition of what is meant by the angles θ_x and θ_y must be made more explicit. The most reasonable way to do this is to treat θ_x as a longitude and θ_y as a latitude. This is demonstrated in Fig. 4.

The most straightforward equivalent to z in the helioprojective system would be d. This does, however, have the slight drawback of making this a left-handed coordinate system. If a right-handed coordinate system is desired, then this can be accomplished by defining the variable ζ as

$$\zeta \equiv D_\odot - d. \tag{5}$$

Surfaces of constant ζ would be concentric spheres centered on the observer, with the sphere representing $\zeta = 0$ passing through the center of the Sun. Positive values of ζ would represent points closer in to the observer than Sun center, while negative values would represent points farther out than Sun center. Close to the Sun, $\zeta \approx z$.

4.2. Helioprojective-Radial Coordinates

This is the helioprojective equivalent of heliocentric-radial, where the impact parameter ρ is replaced with the angle θ_ρ. Again, this is a spherical coordinate system, and the map projection rules come into play. Close to the Sun, where the small-angle approximation holds, the heliocentric-radial and helioprojective-radial impact parameters are related through the equation

$$\rho \approx d(\frac{\pi}{180^\circ})\theta_\rho \approx D_\odot(\frac{\pi}{180^\circ})\theta_\rho. \tag{6}$$

Note that $\theta_\rho = 0$ at one pole of the helioprojective-radial system, where most spherical coordinate systems have $\theta = \pm 90^\circ$ at the poles. We resolve this by defining the declination parameter

$$\delta_\rho \equiv \theta_\rho - 90^\circ. \tag{7}$$

To satisfy the requirements of WCS coordinate transform formalism, angular coordinates in FITS files will be expressed as (δ_ρ, ψ) pairs. The conversion to (θ_ρ, ψ) is then accomplished simply as $\theta_\rho = \delta_\rho + 90^\circ$.

4.2.1. Computing the Surface Normal

A variation on radial coordinates involves replacing either ρ or θ_ρ with the cosine of the angle between the surface normal and the line of sight to the observer. This parameter, μ, varies between 1 at disk center to 0 at the limb. Because this describes spatial positions at the Sun's surface, it has the same limitations as heliographic coordinates. (See Sec. 2.)

The conversion between heliocentric coordinates and μ is quite simple

$$\mu = \sqrt{1 - (\rho/R_\odot)^2}. \tag{8}$$

On the other hand, the conversion between helioprojective coordinates and μ is more complicated, and is given by

$$\mu = \frac{1 - ab}{\sqrt{(1 + a^2)(1 + b^2)}}, \tag{9}$$

where

$$\begin{aligned} a &= \tan\theta_\rho, \\ q &= \sqrt{R_\odot^2(1 + a^2) - a^2 D_\odot^2}, \\ b &= \frac{a(D_\odot - q)}{a^2 D_\odot + q}. \end{aligned}$$

However, when $D_\odot \gg R_\odot$, the distinction between Eqs. (8) and (9) is negligible. Only when $D_\odot$ is within 8 solar radii does the error grow to 0.1%, and at 1 A.U. the approximation is good to almost six significant figures. Thus, for most applications, Eq. (9) can be simplified to

$$\mu \approx \sqrt{1 - (\theta_\rho/\theta_{\rho_{\rm limb}})^2}. \tag{10}$$

5. World Coordinate System Implementations

Table 2 lists all the coordinate parameters referred to in this document, together with their units and WCS labels, where appropriate. The WCS labels are designed to be used in `CTYPE`*i* declarations, e.g. `CTYPE1='HPLN-TAN'`. They also serve as the basis for a family of keywords establishing the viewer's location (see Sec. 9.1). Spherical coordinates, i.e. heliographic or helioprojective, are always in pairs of the form *xx*LN/*xx*LT, for longitude and latitude, which is why some quantities in Table 2 have synonyms.

Map projections do not enter into heliocentric coordinates, and therefore the axis labels will simply be the appropriate

Table 2. A complete list of all coordinate parameters referred to in this document. Also included is the associated WCS label where appropriate, and the units in which the parameters are expressed. Parameters given in meters can also be expressed as relative to the solar radius $R_\odot$.

Par	Units	WCS label	Description
t	-		Observation date/time
$R_\odot$	m		Solar radius ($\approx 6.96 \times 10^8$ m)
$D_\odot$	m	`DSUN`	Distance between the observer and Sun center
Θ	deg	`HGLT`	Heliographic latitude
		`CRLT`	Synonym when used with `CRLN`
Φ	deg	`HGLN`	Stonyhurst heliographic longitude
Φ_C	deg	`CRLN`	Carrington heliographic longitude
L_0	deg		Carrington longitude of central meridian as seen from Earth
r	m	`HECR`	Radial distance from sun center
h	m	`HECH`	Radial distance from the solar surface, $h = r - R_\odot$
x	m	`SOLX`	Heliocentric westward distance
y	m	`SOLY`	Heliocentric northward distance
z	m	`SOLZ`	Heliocentric observerward distance
ρ	m	`SOLI`	Heliocentric impact parameter
ψ	deg	`HCPA`	Position angle from solar North
		`HRLN`	Synonym when used with `HRLT`
θ_x	deg	`HPLN`	Helioprojective westward angle
θ_y	deg	`HPLT`	Helioprojective northward angle
θ_ρ	deg		Helioprojective impact angle
δ_ρ	deg	`HRLT`	Helioprojective-radial declination angle, defined as $\theta_\rho - 90°$
d	m	`DIST`	Radial distance from the observer
ζ	m	`HPRZ`	Helioprojective analog of z, defined as $D_\odot - d$
μ	-		Cosine of the angle to the surface normal
B_0	deg		Tilt of the solar North rotational axis toward the observer (heliographic latitude of the observer)
Φ_0	deg		Stonyhurst heliographic longitude of the observer

four-letter designation as given in Table 2. The `CUNIT`*i* keyword will give the units of the coordinate values. Some examples of valid entries for `CUNIT`*i* are

```
CUNIT1  = 'm       '  /meter
CUNIT1  = 'km      '  /kilometer
CUNIT1  = 'Mm      '  /megameter
CUNIT1  = 'solRad  '  /solar radius
```

See Greisen & Calabretta (2002) for more information.

The heliographic and helioprojective coordinate systems, however, are spherical in nature, and therefore are subject to the map projection formalism described in Calabretta & Greisen (2002). Some of the more common projections used are discussed below.

5.1. `TAN`: *Gnomonic Projection*

One common observing method is where a solar image is focused onto a flat focal plane, such as a CCD detector. The data can then be described as being in helioprojective-cartesian coordinates. In this case, the gnomonic projection, or `TAN`, comes into play. This will place the following requirements on the WCS keywords:

- `CRPIX`*j*: Will express the pixel coordinates of the on-axis point. Often, the center of the array, or solar disk center, is a reasonable approximation.
- `CDELT`*i*: Will scale the coordinates into degrees.
- `CTYPE`*i*: The values for the (θ_x, θ_y) coordinate axes will be `'HPLN-TAN'` and `'HPLT-TAN'` respectively.
- `CRVAL`*i*: The coordinates of the reference pixel, in degrees.
- `LONPOLE`: Normally 180°. See Calabretta & Greisen (2002) for a complete discussion.

Most observations using the helioprojective coordinate system will use the `TAN` projection.

The coordinate axes should be defined so that the associated parameter is locally increasing. There is, however, an ambiguity in the helioprojective-radial case when the reference pixel is disk center. At disk center, δ_ρ increases in all directions, and the value of ψ becomes indeterminate. In that case, the δ_ρ axis should point toward the solar North pole, while the ψ axis should point toward the East limb. The reference value for disk center will be given as $(\delta_\rho, \psi) = (-90°, 0)$. With these definitions, the value of `LONPOLE` will be 180°.

5.2. `AZP`: *Perspective Zenithal Projection*

Heliographic coordinates can be described as a perspective zenithal projection, or `AZP`. The keywords in this case would be

- `CRPIX`*j*: Normally, the pixel coordinates of disk center, even if that's outside the array. (However, see Calabretta & Greisen (2002) for a discussion of slant zenithal projections.)
- `CDELT`*i*: For the `AZP` projection, as in the `TAN` projection, the coordinates will be scaled into degrees. However, because of the geometry of the `AZP` projection, these degrees will be larger than the angular extent on the sky by a factor of $(D_\odot/R_\odot - 1)$. At a distance of 1 A.U., this renormalization factor is approximately 213.9.
- `CTYPE`*i*: The `CTYPE`*i* keyword for the longitude dimension will be either `'HGLN-AZP'` or `'CRLN-AZP'`, depending on whether Stonyhurst or Carrington heliographic longitudes are used, while the value for the latitude dimension will be either `'HGLT-AZP'` or `'CRLT-AZP'` respectively.
- `PV`*i*`_1`: This keyword will contain the value $-D_\odot/R_\odot$ (note the minus sign), where *i* is the index of the latitude coordinate axis. At a distance of 1 A.U., this is approximately -214.9.
- `CRVAL`*i*: These will specify the latitude and longitude of the reference (disk center) pixel in degrees. Thus, together with the `PV`*i*`_1` keyword, the information about the observer's position is complete.

LONPOLE: Unless one is looking straight down along one of the poles, this should normally be 180°.

5.3. SIN: Orthographic Projection

A special case of the AZP projection is when $D_\odot$ is large enough to be considered to be infinitely far away. This is known as the SIN projection and is implemented in FITS headers as follows:

- CRPIX*j*: The pixel coordinates of disk center, even if that's outside the array.
- CDELT*i*: The plate scale to be used for the SIN projection is $180°/\pi$ times the plate scale in solar radii per pixel.
- CTYPE*i*: The CTYPE*i* keyword for the longitude dimension will be either 'HGLN-SIN' or 'CRLN-SIN', while the value for the latitude dimension will be either 'HGLT-SIN' or 'CRLT-SIN' respectively.
- PV*i*_1: Set to 0, where *i* is the latitude axis.
- PV*i*_2: Set to 0, where *i* is the latitude axis.
- CRVAL*i*: These will specify the latitude and longitude of the reference (disk center) pixel in degrees.
- LONPOLE: Same as for the AZP projection.

For most cases, the AZP projection is preferable to the SIN projection, but the latter can be a useful approximation when not all the information for the AZP projection is available.

5.4. CAR: Plate Carrée Projection

Cylindrical projections are well suited for synoptic maps, where observations are made over a period of time to build up a complete rotation's worth of data. Either Stonyhurst or Carrington heliographic coordinates can be used. The simplest kind of cylindrical projection is plate carrée (CAR), where both the longitude and latitude values are equally spaced in degrees. In order for the cylindrical axis to be aligned with the solar rotation axis, the reference pixel must be on the equator. Otherwise, the values of CRPIX*j*, CRVAL*i*, and CDELT*i* are straightforward. The CTYPE*i* keyword for the longitude dimension will be either 'HGLN-CAR' or 'CRLN-CAR', while the value for the latitude dimension will be either 'HGLT-CAR' or 'CRLT-CAR' respectively. The default value of LONPOLE is zero.

We propose that the additional keyword CAR_ROT be used to associate the appropriate Carrington rotation number with the observation. For example, a synoptic map covering the period beginning on 2 September 1995 would have CAR_ROT = 1900. For a map containing data beginning within one Carrington rotation and ending within the next, the value of CAR_ROT will be associated with the reference pixel given by the CRPIX*j* keywords.

Care should be used in the formulation and use of synoptic maps, which are generally built up from data taken over a period of weeks. Because the Sun rotates differentially, the Carrington longitude of any feature will evolve over time, regardless of any evolution of the feature itself. The observation time will vary from one part of the map to another, and will depend on how the map was constructed. The general problem of mapping synoptic coordinates into instantaneous spatial coordinates is too complex for the present work. Synoptic map data providers are encouraged to document in the FITS header how the map was constructed.

Another use of the plate carrée projection is when making maps of the corona, where one axis is position angle, and the other is the radial distance from disk center. This can be naturally handled either as heliocentric coordinates in physical units (HCPA/SOLI), or as angular helioprojective-radial coordinates in the plate carrée projection (HRLN-CAR/HRLT-CAR).

5.5. CEA: Cylindrical Equal Area Projection

Another commonly used projection for synoptic maps is cylindrical equal area (CEA), where the latitude pixels are equally spaced in the sine of the angle. In its simplest form, the keywords for the *latitude* axis *i* are

- CDELT*i*: Set to $180°/\pi$ times the pixel spacing of the sine of the latitude.
- PV*i*_1: Set to 1. (See Calabretta & Greisen (2002) for a discussion of the more general case, where PV*i*_1 < 1.)
- CTYPE*i*: Either 'HGLT-CEA' or 'CRLT-CEA', while the same for the longitude axis will be either 'HGLN-CEA' or 'CRLN-CEA' respectively.

The other keywords are the same as for the CAR projection, including LONPOLE=0, and the proviso that the reference pixel be a point on the equator, which is true for all cylindrical and pseudo-cylindrical projections.

6. Sample FITS Headers

Figure 5 shows a sample header for a hypothetical instrument with a plate scale of 3.6 arcsec/pixel, observing from 1 A.U. The detector is a 1024×1024 CCD, with the Sun centered in the CCD. Three different coordinate systems are shown: heliocentric-cartesian, helioprojective-cartesian, and Stonyhurst heliographic. Note that the pixel scale for the heliographic coordinates is $D_\odot/R_\odot - 1 = 213.9$ times the pixel scale for the helioprojective coordinates. Although omitted in the FITS header here to save space, the default value of LONPOLE=180° is used in both the helioprojective and heliographic coordinate systems. In general, it's recommended that all keywords be included in the header, even when they take default values.

Helioprojective-radial coordinates are demonstrated in Fig. 6. In this example, a scanning slit spectrometer is used to scan the corona. The slit is oriented parallel to the limb, and is scanned outward along a radial at position angle -45°, starting at 18 arcmin ($0\overset{\circ}{.}3$) from disk center. Note that the position angle ψ is denoted as 'HRLN-TAN' because it is here part of the helioprojective coordinate system. When used as part of a heliocentric coordinate system, it simply denoted as 'HCPA'. Note also that δ_ρ is used instead of θ_ρ, so the radial position is given as $-89\overset{\circ}{.}7$ rather than $0\overset{\circ}{.}3$. Although not included in the FITS header, the default value of LONPOLE=180° is used in both helioprojective coordinate systems.

```
NAXIS   =                    2 /2D image
NAXIS1  =                 1024 /Number of columns
NAXIS2  =                 1024 /Number of rows

WCSNAME = 'Heliocentric-cartesian (approximate)'

CTYPE1  = 'SOLX    '           /Axis labels
CTYPE2  = 'SOLY    '           /
CRPIX1  =                512.5 /Center of CCD
CRPIX2  =                512.5 /
CUNIT1  = 'solRad  '           /Solar radii
CUNIT2  = 'solRad  '           /
CDELT1  =              0.00375 /[radii] Plate scale
CDELT2  =              0.00375 /
CRVAL1  =                  0.0 /Sun center
CRVAL2  =                  0.0 /

WCSNAMEA= 'Helioprojective-cartesian'

CTYPE1A = 'HPLN-TAN'           /Axis labels (ThetaX)
CTYPE2A = 'HPLT-TAN'           /            (ThetaY)
CRPIX1A =                512.5 /Center of CCD
CRPIX2A =                512.5 /
CUNIT1A = 'deg     '           /Angles in degrees
CUNIT2A = 'deg     '           /
CDELT1A =                0.001 /[deg] Plate scale
CDELT2A =                0.001 /
CRVAL1A =                  0.0 /Sun center
CRVAL2A =                  0.0 /

WCSNAMEB= 'Stonyhurst heliographic'

CTYPE1B = 'HGLN-AZP'           /Heliogr. longitude
CTYPE2B = 'HGLT-AZP'           /         latitude
CRPIX1B =                512.5 /Center of CCD
CRPIX2B =                512.5 /
CUNIT1B = 'deg     '           /Angles in degrees
CUNIT2B = 'deg     '           /
CDELT1B =               0.2139 /0.001*(Dsun/Rsun-1)
CDELT2B =               0.2139 /
CRVAL1B =                  0.0 /Central meridian
CRVAL2B =                  6.5 /B0 angle
PV2_1B  =               -214.9 /Negative Dsun/Rsun
```

Fig. 5. Sample FITS header, demonstrating heliocentric-cartesian, helioprojective-cartesian, and Stonyhurst heliographic coordinates for the same array. For simplicity, not all FITS keywords are shown

As noted earlier, a special case of the use of the helioprojective-radial coordinate system is when the reference pixel is disk center. This case is illustrated in Fig. 7 for the same observation as in Fig. 5. Note that `CDELT1C` is the negative of the value `CDELT1A` shown in Fig. 5, because the ψ unit vector as defined points toward the East limb. Consult Calabretta & Greisen (2002) for more information about how the parameters are used to define the coordinate system.

7. Coordinate Conversions

The following equations describe the conversion from one coordinate system into another, and are intended to demonstrate the relationships between the coordinate systems. Other coordinate conversions not shown below can be derived from those shown. Where appropriate, the assumptions to be made when one of the dimensions is missing is discussed. When converting between heliocentric and helioprojective coordinates, if all three spatial dimensions are not present, and no additional constraints (such as $r = R_\odot$) can be applied, then the small angle approximation may be used instead. (See Eqs. (4) and (6).)

```
NAXIS   =                    3 /3D array
NAXIS1  =                 1024 /No. wavelengths
NAXIS2  =                  100 /No. slit pixels
NAXIS3  =                  100 /No. slit exposures

WCSNAME = 'Helioprojective-radial'

CTYPE1  = 'WAVE    '           /Wavelength
CTYPE2  = 'HRLN-TAN'           /Position angle
CTYPE3  = 'HRLT-TAN'           /rad. dist. - 90 deg
CRPIX1  =                512.5 /Center of CCD
CRPIX2  =                 50.5 /Slit center
CRPIX3  =                  1.0 /First exposure
CUNIT1  = 'Angstrom'           /Wavelenth in Ang.
CUNIT2  = 'deg     '           /Angles in degrees
CUNIT3  = 'deg     '           /
CDELT1  =                  0.1 /Wavelength scale
CDELT2  =                0.001 /Slit pixel size
CDELT3  =                0.001 /Scan step size
CRVAL1  =                350.0 /Reference wavelength
CRVAL2  =                -45.0 /Pos. angle slit cntr
CRVAL3  =                -89.7 /Rad. dist. 18 arcmin

WCSNAMEA= 'Helioprojective-cartesian'

CTYPE1A = 'WAVE    '           /Wavelength
CTYPE2A = 'HPLT-TAN'           /Theta_Y
CTYPE3A = 'HPLN-TAN'           /Theta_X
CRPIX1A =                512.5 /Center of CCD
CRPIX2A =                 50.5 /Slit center
CRPIX3A =                  1.0 /First exposure
CUNIT1A = 'Angstrom'           /Wavelength in Ang.
CUNIT2A = 'deg     '           /Angles in degrees
CUNIT3A = 'deg     '           /
CDELT1A =                  0.1 /Wavelength scale
CDELT2A =                0.001 /Scale after rotation
CDELT3A =                0.001 /
PC1_1A  =                  1.0 /No wavelength trans.
PC2_2A  =             0.707107 /Rotates by 45 deg.
PC2_3A  =             0.707107 /
PC3_2A  =            -0.707107 /
PC3_3A  =             0.707107 /
CRVAL1A =                350.0 /Reference wavelength
CRVAL2A =             0.212132 /Coord. of ref. pixel
CRVAL3A =             0.212132 /
```

Fig. 6. Sample FITS header, demonstrating helioprojective-radial, and helioprojective-cartesian coordinates for the same array. See text for details. For simplicity, not all FITS keywords are shown.

```
WCSNAMEC= 'Helioprojective-radial'

CTYPE1C = 'HRLN-TAN'           /Position Angle
CTYPE2C = 'HRLT-TAN'           /Delta_Rho
CRPIX1C =                512.5 /Center of CCD
CRPIX2C =                512.5 /
CUNIT1C = 'deg     '           /Angles in degrees
CUNIT2C = 'deg     '           /
CDELT1C =               -0.001 /Lat.  -> E (deg/pix)
CDELT2C =                0.001 /Long. -> N (deg/pix)
CRVAL1C =                  0.0 /Ref. latitude is 0
CRVAL2C =                -90.0 /Sun center
LONPOLEC=                180.0 /Default value
```

Fig. 7. Sample portion of a FITS header, demonstrating helioprojective-radial coordinates for the same array shown in Fig. 5.

Between Stonyhurst heliographic and heliocentric-cartesian:

$$\begin{aligned} x &= r\cos\Theta\sin(\Phi-\Phi_0), \\ y &= r[\sin\Theta\cos B_0 - \cos\Theta\cos(\Phi-\Phi_0)\sin B_0], \\ z &= r[\sin\Theta\sin B_0 + \cos\Theta\cos(\Phi-\Phi_0)\cos B_0], \end{aligned} \tag{11}$$

$$\begin{aligned} r &= \sqrt{x^2+y^2+z^2}, \\ \Theta &= \sin^{-1}((y\cos B_0 + z\sin B_0)/r), \\ \Phi &= \Phi_0 + \arg(z\cos B_0 - y\sin B_0, x), \end{aligned} \tag{12}$$

where B_0 and Φ_0 are the Stonyhurst heliographic latitude and longitude of the observer. If the r dimension is missing, then we make the assumption that $r = R_\odot$. If the z dimension is missing, then we again make the assumption that $r = R_\odot$, and therefore $z = \sqrt{R_\odot^2 - x^2 - y^2}$.

Between heliocentric-cartesian and heliocentric-radial:

$$\begin{aligned} \rho &= \sqrt{x^2+y^2}, \\ \psi &= \arg(y, -x), \\ z &= z, \end{aligned} \tag{13}$$

$$\begin{aligned} x &= -\rho\sin\psi, \\ y &= +\rho\cos\psi, \\ z &= z. \end{aligned} \tag{14}$$

Between helioprojective-cartesian and heliocentric-cartesian:

$$\begin{aligned} x &= d\cos\theta_y\sin\theta_x, \\ y &= d\sin\theta_y, \\ z &= D_\odot - d\cos\theta_y\cos\theta_x, \end{aligned} \tag{15}$$

$$\begin{aligned} d &= \sqrt{x^2+y^2+(D_\odot - z)^2}, \\ \theta_x &= \arg(D_\odot - z, x), \\ \theta_y &= \sin^{-1}(y/d). \end{aligned} \tag{16}$$

Between helioprojective-radial and heliocentric-radial:

$$\begin{aligned} \rho &= d\sin\theta_\rho, \\ \psi &= \psi, \\ z &= D_\odot - d\cos\theta_\rho, \end{aligned} \tag{17}$$

$$\begin{aligned} \theta_\rho &= \arg(D_\odot - z, \rho), \\ \psi &= \psi, \\ d &= \sqrt{\rho^2 + (D_\odot - z)^2}. \end{aligned} \tag{18}$$

7.1. Converting to Space Physics Coordinates

The simplest relationship between the solar imaging coordinate systems discussed above and the coordinate systems used in space physics is that between Heliocentric Cartesian coordinates and Heliocentric Earth Equatorial (HEEQ) coordinates (Hapgood 1992). In HEEQ coordinates, the $Z_{\rm HEEQ}$ axis is aligned with the solar North rotation pole, while the $X_{\rm HEEQ}$ axis points toward the intersection between the solar equator and the solar central meridian as seen from Earth. The conversion between Heliocentric Cartesian (x, y, z) as seen from Earth, and Heliocentric Earth Equatorial $(X_{\rm HEEQ}, Y_{\rm HEEQ}, Z_{\rm HEEQ})$ is then

$$\begin{aligned} X_{\rm HEEQ} &= z\cos B_0 - y\sin B_0, \\ Y_{\rm HEEQ} &= x, \\ Z_{\rm HEEQ} &= z\sin B_0 + y\cos B_0, \end{aligned} \tag{19}$$

$$\begin{aligned} x &= Y_{\rm HEEQ}, \\ y &= Z_{\rm HEEQ}\cos B_0 - X_{\rm HEEQ}\sin B_0, \\ z &= Z_{\rm HEEQ}\sin B_0 + X_{\rm HEEQ}\cos B_0, \end{aligned} \tag{20}$$

where B_0 is the tilt of the solar North rotation axis toward Earth. For further conversion to other space physics coordinate systems, see Russell (1971), Hapgood (1992), and Fränz & Harper (2002).

7.2. Pseudo-angles and the `TAN` projection

Solar astronomers have not traditionally used spherical trigonometry for image coordinates, except in the heliographic case. Instead, it's more common to work in pseudo-angles representing the plate scale of the instrument. These pseudo-angles vary as the tangent of the actual angles. This is identical to working in the gnomonic (`TAN`) projection. Close to the Sun, the pseudo-angles are approximately equal to their real angle counterparts. (From 1 A.U., on the solar disk, they're the same to at least five significant figures.)

The relationship between the pseudo-angle ρ_θ and the actual angle θ_ρ is fairly simple,

$$\rho_\theta = \left(\frac{180^\circ}{\pi}\right)\tan\theta_\rho, \tag{21}$$

$$\theta_\rho = \tan^{-1}\left[\left(\frac{\pi}{180^\circ}\right)\rho_\theta\right], \tag{22}$$

while the relationships for helioprojective-cartesian coordinates are somewhat more complicated,

$$x_\theta = \left(\frac{180^\circ}{\pi}\right)\tan\theta_x, \quad (23)$$
$$y_\theta = \left(\frac{180^\circ}{\pi}\right)\frac{\tan\theta_y}{\cos\theta_x},$$

$$\theta_x = \tan^{-1}\left[\left(\frac{\pi}{180^\circ}\right)x_\theta\right], \quad (24)$$
$$\theta_y = \tan^{-1}\left[\left(\frac{\pi}{180^\circ}\right)\frac{y_\theta}{\sqrt{1+(\pi x_\theta/180^\circ)^2}}\right].$$

(In the above we assume that the pseudo-angles ρ_θ, x_θ, and y_θ are all expressed in pseudo-degrees; hence the conversion factors of $180^\circ/\pi$. Additional conversion factors would be needed for pseudo-arcminutes or pseudo-arcseconds. We also assume that the optical axis is aligned with Sun center, to simplify the discussion.) Although the conversion between pseudo-angles and true angles is not necessarily simple, pseudo-angles are much easier to deal with than true angles, and act more like heliocentric coordinates. For example, in pseudo-angles, the relationships between helioprojective-cartesian and helioprojective radial coordinates are the same as their heliocentric counterparts:

$$\rho_\theta = \sqrt{{x_\theta}^2 + {y_\theta}^2}, \quad (25)$$
$$\psi = \arg(y_\theta, -x_\theta),$$

$$x_\theta = -\rho_\theta \sin\psi, \quad (26)$$
$$y_\theta = +\rho_\theta \cos\psi.$$

The approximations of Eqs. (4) and (6) work equally well with the pseudo angles x_θ, y_θ, ρ_θ as with their real-angle counterparts. In fact, the distances

$$x' = D_\odot\left(\frac{\pi}{180^\circ}\right)x_\theta,$$
$$y' = D_\odot\left(\frac{\pi}{180^\circ}\right)y_\theta, \quad (27)$$
$$\rho' = D_\odot\left(\frac{\pi}{180^\circ}\right)\rho_\theta,$$

are true distances of the projection of a point onto the $z = 0$ plane.

As noted before, the pseudo angles (x_θ, y_θ) are equivalent to the true helioprojective angles (θ_x, θ_y) as implemented in a `TAN` projection. In fact, these are what would be calculated from the WCS keywords if the spherical transformation implied by the letters `-TAN` in the `CTYPE`*i* keywords was not applied. We anticipate that this is how the helioprojective-cartesian coordinates with the `TAN` projection in FITS files will often be implemented within the solar community. In other words, the coordinates will be calculated from the `CRPIX`*j*, `CRVAL`*i*, `CDELT`*i*, `PC`*i*_*j* (or `CROTA`*i*) keywords in the normal fashion, with the result being the pseudo-angles described above. We believe that this is a proper approach for most data analysis, so long as the underlying spherical nature of the coordinates can be retrieved when necessary. To that end, the coordinates should be marked in the FITS header as being in the `TAN` projection, even though this will not normally affect how the coordinates are read in and applied.

The situation is somewhat different with helioprojective-radial coordinates, which require the full WCS spherical formalism to make sense of the information in the FITS headers when the `TAN` projection is used.

8. The older FITS coordinate system

Over the last few years, an informal standard has been developing for solar image coordinates in FITS files, using the older FITS keywords rather than the newer WCS formalism. This system has the following characteristics:

- The coordinate axes correspond to Fig. 2.
- The coordinate axis labels vary, or are omitted, but are typically `SOLARX`, `SOLARY`, as used by the SOHO project.
- The coordinates are usually expressed in arcseconds (although the appropriate `CUNIT`*i* keywords may not appear in the header). However, no map projections are explicitly given, and the distinction between heliocentric and helioprojective coordinates is glossed over.

This system can be incorporated into the overall formalism being considered here by recognizing that the coordinates given in such a system are actually the pseudo-angles discussed in Sec. 7.2. Thus, this system is implicitly helioprojective-cartesian with the standard `TAN` projection.

Although we encourage the eventual adoption of a WCS-based system, FITS readers should also be written to parse files using the older coordinate system described above, as an implicit helioprojective-cartesian system. This format is acceptable for simple two-dimensional solar images. FITS files using this older system should include the keywords `CRPIX`*j*, `CRVAL`*i*, `CDELT`*i*, `CTYPE`*i*, `CUNIT`*i*, and optionally `CROTA`*i* if the image is rotated. When `CROTA`*i* is included, then the conversion from pixel coordinates (i, j) to spatial coordinates (x, y) should be done via the following equations (Calabretta & Greisen 2002).

$$\gamma = \mathtt{CROTA}j,$$
$$x = \mathtt{CDELT}i \times \cos\gamma \times i - \mathtt{CDELT}j \times \sin\gamma \times j, \quad (28)$$
$$y = \mathtt{CDELT}i \times \sin\gamma \times i + \mathtt{CDELT}j \times \cos\gamma \times j,$$

where i is the x-like axis, and j is the y-like axis. This is equivalent in the WCS to a `CD` matrix with the following definition:

$$\mathtt{CD}i\_i = +\mathtt{CDELT}i \times \cos(\mathtt{CROTA}j),$$
$$\mathtt{CD}i\_j = -\mathtt{CDELT}j \times \sin(\mathtt{CROTA}j), \quad (29)$$
$$\mathtt{CD}j\_i = +\mathtt{CDELT}i \times \sin(\mathtt{CROTA}j),$$
$$\mathtt{CD}j\_j = +\mathtt{CDELT}j \times \cos(\mathtt{CROTA}j),$$

or to a `PC` matrix with the following definition:

$$\mathtt{PC}i\_i = +\cos(\mathtt{CROTA}j),$$
$$\mathtt{PC}i\_j = -\sin(\mathtt{CROTA}j) \times (\mathtt{CDELT}j/\mathtt{CDELT}i), \quad (30)$$
$$\mathtt{PC}j\_i = +\sin(\mathtt{CROTA}j) \times (\mathtt{CDELT}i/\mathtt{CDELT}j),$$
$$\mathtt{PC}j\_j = +\cos(\mathtt{CROTA}j).$$

Table 3. Comparison of the FITS coordinate keywords in the old and new system, for helioprojective-cartesian coordinates in the `TAN` projection. The phrase "exactly the same" assumes that the same units are used in both cases. See Sec. 9.2 for a discussion of angular units. Although `CUNIT`*i* did not appear in the original Wells et al. (1981) paper, it has been commonly used.

Keyword	Original	WCS
`CRPIX`*j*	Exactly the same in both systems	
`CRVAL`*i*	Exactly the same in both systems	
`CDELT`*i*	Exactly the same in both systems	
`CROTA`*i*	Rotation angle	Obsolete
`PC`*i*_*j*	Not used	Replaces `CROTA`*i*
`CUNIT`*i*	Exactly the same in both systems	
`CTYPE`*i*	`SOLARX`	`HPLN-TAN`
	`SOLARY`	`HPLT-TAN`

(The inverse problem of decomposing a `PC` matrix into `CROTA`*i* values, and how to determine if such a decomposition is possible, is discussed in Calabretta & Greisen (2002).) Table 3 shows the usage of the various FITS coordinate system keywords in the old and new systems.

There has also recently been growing usage of some metadata keywords for solar images, such as `XCEN`, `YCEN`, and `ANGLE`. Although these are very useful keywords for cataloging sets of observations, they should *never* be considered as substitutes for the standard FITS coordinate system keywords. However, these can also be incorporated by making the identifications

$$\begin{aligned}
\mathtt{CRPIX1} &= (\mathtt{NAXIS1}+1)/2, \\
\mathtt{CRPIX2} &= (\mathtt{NAXIS2}+1)/2, \\
\mathtt{CRVAL1} &= \mathtt{XCEN}, \\
\mathtt{CRVAL2} &= \mathtt{YCEN}, \\
\mathtt{CROTA1} &= \mathtt{ANGLE}, \\
\mathtt{CROTA2} &= \mathtt{ANGLE}.
\end{aligned} \tag{31}$$

9. Additional Considerations

9.1. Observer's Position

As mentioned earlier, some of the coordinate systems mentioned here are oriented with one of the coordinate axes pointing toward the observer. Therefore, information should be placed in the FITS header to specify the viewer's location. This is done by using the labels listed in Table 2 followed by the characters "`_OBS`". Since there's no standard mechanism for specifying what units that the value of a keyword in the header takes, all distance keywords will have the units *meters*, while angular keywords are in *degrees*. In principal, any standard coordinate system could be used to specify the position of the observer, but it is recommended that at least the keywords `DSUN_OBS`, `HGLN_OBS`, and `HGLT_OBS` be included to specify the Stonyhurst heliographic coordinates of the observer. For example, a FITS header might include the lines

```
DSUN_OBS=          1.507E+11
HGLN_OBS=                0.0
HGLT_OBS=               7.25
```

If the observer's position is not specified, then it should be assumed that the observation was made from Earth, or from low Earth orbit.

The observer's position can also be specified in any of the standard space physics coordinate systems (Russell 1971; Hapgood 1992; Fränz & Harper 2002). Table 4 lists these coordinate systems, together with their associated WCS labels. Like the other keywords discussed in this document, the observer's position would be specified in the header by appending the characters "`_OBS`", e.g. `HEQX_OBS, HEQY_OBS, HEQZ_OBS`. In keeping with WCS standards, the values of these keywords would be in *meters*.

Table 4. Standard space physics coordinate systems from Russell [1971], Hapgood [1992], and Fränz and Harper [2002], together with their associated WCS labels for `CTYPE`*i* declarations.

Coordinate system	Abbrev.	WCS labels
Geocentric equatorial inertial	GEI	`GEIX, GEIY, GEIZ`
Geographic	GEO	`GEOX, GEOY, GEOZ`
Geocentric solar ecliptic	GSE	`GSEX, GSEY, GSEZ`
Geocentric solar magnetic	GSM	`GSMX, GSMY, GSMZ`
Solar magnetic	SM	`SM_X, SM_Y, SM_Z`
Geomagnetic	MAG	`MAGX, MAGY, MAGZ`
Helicentric Aries Ecliptic	HAE	`HAEX, HAEY, HAEZ`
Heliocentric Earth Ecliptic	HEE	`HEEX, HEEY, HEEZ`
Heliocentric Earth Equatorial	HEEQ	`HEQX, HEQY, HEQZ`
Heliocentric Intertial	HCI	`HCIX, HCIY, HCIZ`

Table 5. Alternate angular units.

Unit String	Meaning	Notes
`deg`	degree of arc	$\pi/180$ radians
`arcmin`	minute of arc	1/60 degree
`arcsec`	second of arc	1/3600 degree
`mas`	millisecond of arc	1/3600000 degree
`rad`	radian	$180/\pi$ degrees

9.2. Other Angular Units

It has long been standard in FITS files that angular measurements be expressed in degrees (Wells et al. 1981; Calabretta & Greisen 2002). However, it is also common in solar observations to express the coordinates in arcseconds from disk center. This possibility was presaged in Sec. 7.2, where helioprojective-cartesian pseudo-angles can be calculated from the FITS header parameters without resorting to spherical coordinate calculations, which is not the case for helioprojective-radial coordinates. Thus, *only* for the limited case where the coordinate axes are `HPLN-TAN` and `HPLT-TAN`, and where the coordinates are all relatively close to the Sun, it can be appropriate to express the coordinates in units other than degrees. Although strongly discouraged, the World Coordinate System (Greisen & Calabretta 2002) does allow for this possibility through the `CUNIT`*i* keyword, which can take on the values given in Table 5. For example, if units of arcseconds is desired for a two dimensional image array, then this could be specified by setting

```
CTYPE1  = 'HPLN-TAN'
```

```
CUNIT1  = 'arcsec  '
CTYPE2  = 'HPLT-TAN'
CUNIT2  = 'arcsec  '
```

This allows for compatibility with other proposed standards for solar image data, which have generally called for coordinates to be expressed in arcseconds. Note that the value of the CUNITi keyword *only* applies to the keywords CRVALi, and CDELTi (or CDi_j). All other angular keywords, such as LONPOLE, CROTAi, etc., *must* be in degrees.

9.3. Celestial coordinates

Although the helioprojective coordinates described here are similar in many ways to celestial coordinates, they have not been defined to be applicable to any other bodies than the Sun and its atmosphere. A solar data set may also contain other astronomical bodies of interest, such as sun-grazing comets or calibration stars. Analysis of these extra-solar targets would be aided by including additional coordinate information in one of the traditional celestial coordinate systems (e.g R.A., Dec.) as discussed in Calabretta & Greisen (2002). This is particularly true when the solar coordinates are expressed in arcseconds, which may cause trouble for software used by non-solar astronomers.

10. Conclusions

We have presented a standardized coordinate system for solar image data which is precise, rigorous, and comprehensive. It builds on current standards, and extends them for the needs of future missions. Both terrestrial and non-terrestrial viewpoints are supported. By incorporating the emerging World Coordinate System standard, compatibility with future astronomy software is ensured, and a powerful level of flexibility is achieved. Many current FITS files are incorporated as a special subset of the new system.

The coordinate systems presented here have the following characteristics:

– Stonyhurst and Carrington heliographic coordinates are defined to be the same for all observers. Two observers with distinctly different perspectives, such as the STEREO spacecraft, would measure the same heliographic coordinates for a feature at $r = R_\odot$.
– Helioprojective coordinates, and the associated heliocentric systems discussed here, are observer-specific. Two observers with different perspectives, such as STEREO, would measure different coordinates for the same feature, because one axis always points towards the observer.
– The keywords HGLN_OBS, HGLT_OBS, and DSUN_OBS specify the position of the observer.
– The keyword CAR_ROT specifies the Carrington rotation number.

We recommend that new solar data be written using the World Coordinate System formalism discussed here. The simplest of the WCS-compliant coordinate systems is the helioprojective-cartesian system in the TAN projection, and is also the best match to current practice for many observatories. A simple way to implement this scheme, when no rotation needs to be applied, is to set

$$
\begin{aligned}
\mathtt{CRPIX}j &= (\mathtt{NAXIS}i + 1)/2 \\
\mathtt{CRVAL}i &= \text{(pointing of center of image in arcseconds)} \\
\mathtt{CDELT}i &= \text{(pixel spacing in arcseconds)} \\
\mathtt{CUNIT}i &= \texttt{'arcsec '} \\
\mathtt{CTYPE1} &= \texttt{'HPLN-TAN'} \\
\mathtt{CTYPE2} &= \texttt{'HPLT-TAN'}
\end{aligned}
$$

Alternatively, the CRPIXj values can represent the position of disk center in the image, and then CRVALi = 0.

Although this proposal treats solar images as a spherical coordinate system, the complete formalism of spherical coordinates is not required for most solar images. So long as the instrument field-of-view is on the order of a few degrees or less, the coordinates can be treated as planar to a high degree of accuracy. Thus, when the helioprojective-cartesian system is used with the TAN projection, FITS reader software can leave the coordinates as pseudo-angles in the TAN projection rather than converting into spherical angles. This is the same way that the coordinates of solar images are currently handled. Files using the older non-WCS system can be treated as being implicitly helioprojective-cartesian with the TAN projection.

However, the spherical coordinate map projection capabilities of the World Coordinate System offer a large amount of flexibility in dealing with data which are not cast in straightforward Cartesian terms. With the appropriate map projections, one can handle not only images, but also synoptic maps in the heliographic system, and maps of the corona in radial distance and position angle.

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Appendix A: Time

Although a complete description of the specification of date and time in FITS files is beyond the scope of this document, we wish to bring the reader's attention to the revised specification for dates in FITS files, which was adopted by the IAU Fits Working Group (IAU-FWG) in November 1997 (Hanisch et al. 2001). This applies not only to the keyword DATE-OBS, but all keywords starting with the letters "DATE". Such keywords can take one of two forms. When only the date is required, these keywords can take the form "CCYY-MM-DD", where "CCYY" is the century and year (i.e. the four-digit year), "MM" is the two-digit month, and "DD" is the two-digit day-of-month. For example,

```
DATE-OBS= '2000-03-27'
```

The time can also be included, separated from the year by the letter "T". In that case, the date keywords take the form

"CCYY-MM-DDThh:mm:ss[.sss...]", where "hh", "mm", and "ss" are the hours, minutes, and seconds respectively, and where the seconds can be expanded to an arbitrary fraction. For example,

```
DATE-OBS= '2000-03-27T22:01:41.123'
```

Unless otherwise specified, the dates and times are assumed to be in Coordinated Universal Time (UTC) (except for `DATE` itself, where no assumptions are made).

Before the IAU-FWG adopted the above convention, the SOHO project adopted a similar convention, using the keyword `DATE_OBS` in place of `DATE-OBS` to avoid conflict with the previous standard. Although the standard adopted by SOHO and later by the IAU-FWG are both ultimately based upon ISO-8601 (ISO 1988), the SOHO `DATE_OBS` keyword was based upon a recommendation by the Consultative Committee for Space Data Systems (CCSDS 1990) which was less restrictive than what the IAU-FWG finally adopted. The SOHO keyword differs from the IAU-FWG specification in the following ways:

– It allowed, and in fact encouraged, the date-time string to end in the letter "Z" to indicate the UTC time-zone. The IAU-FWG decided to not allow this.
– It allowed a variation where the day-of-year was given instead of the month and day, which was also not allowed by the IAU-FWG.

The SOHO project adopted the non-standard `DATE_OBS` keyword to address Y2K issues which the FITS committees had not yet addressed, and to allow for the specification of time as well as date. Now that the FITS standard addresses these same issues, one should consider the SOHO `DATE_OBS` keyword to be deprecated in favor of the standard `DATE-OBS` keyword. For backwards compatibility, both forms are acceptable, but `DATE-OBS` is to be preferred.

Appendix B: Comparison of Heliocentric and Helioprojective Coordinates

Heliocentric and helioprojective coordinates have somewhat different properties when applied to the same dataset. The heliocentric system has the advantage that the coordinates are expressed in physical units, simplifying the analysis. This makes it easier to compare data taken at different distances, such as from Earth versus from the inner (L1) Lagrange point. On the other hand, the helioprojective system is a truer representation of the perspective that the data is actually taken in. Some consideration should be given to the possible errors that the use of heliocentric coordinates give over helioprojective coordinates.

The difference between heliocentric coordinates and helioprojective coordinates is demonstrated in Fig. B.1. Assuming that no information is known about the depth parameter (either z or d), the error we need to consider is in the estimation of the transverse distance ρ. When the data is expressed in a `TAN` projection, as is often the case, the plate scale is really a measurement of the tangent of the angle rather than the angle itself, even though expressed in degrees per pixel. This means that the estimated radial distance ρ (or x, or y) will be exactly correct

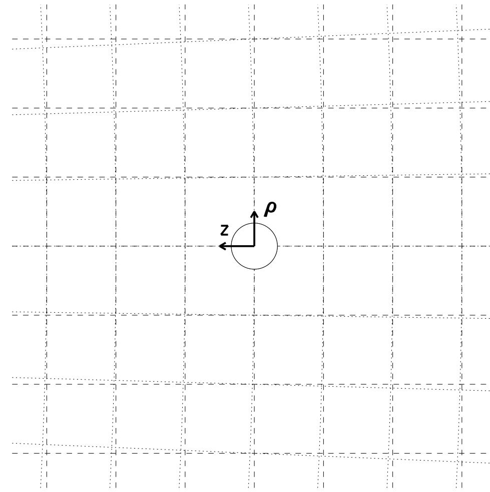

Fig. B.1. Diagram showing the distinction between heliocentric coordinates (dashed lines) and helioprojective coordinates in the `TAN` projection (dotted lines), for an observer located at 1 A.U. to the left of the figure. The full scale of the figure is approximately 20×20 solar radii.

for features located on the $z = 0$ plane. At other locations, the amount of over- or under-estimation is given by

$$\Delta\rho = \frac{\rho z}{D_\odot - z}. \tag{B.1}$$

Since $D_\odot = 214.9 R_\odot$ for an observer at 1 A.U., the errors are on the order of 1% or less for features relatively close to the Sun, and can often be ignored. The potential discrepency grows rapidly, however, at farther distances from the Sun. If we make the assumption that the range of z which may contribute to the line-of-sight intensity scales as ρ, then the possible amount of error scales as ρ^2. The distinction between heliocentric and helioprojective coordinates is therefore important for observations made far out into the corona, particularly for tomographic purposes.

Similarly, the difference between z and ζ is

$$\begin{aligned} z - \zeta &= \sqrt{(D_\odot - z)^2 + \rho^2} - (D_\odot - z) \\ &\approx \frac{1}{2}\left(\frac{\rho^2}{D_\odot - z}\right), \end{aligned} \tag{B.2}$$

which is about half the potential error in ρ, and scales similarly.

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